

A Local Discontinuous Galerkin Discretization for Incompressible Hydrostatic Flows with Free Surface Using σ -Transformed Coordinates

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Introduction

Local Discontinuous Galerkin Method

- *Discontinuous Galerkin (DG) Methods* are a class of Finite Element methods with only elementwise continuous ansatz functions. At discontinuities numerical fluxes are applied.
- The *Local Discontinuous Galerkin (LDG) Methods* are a generalization of the DG methods for advection-diffusion equations

$$\partial_t \mathbf{u} + \nabla \cdot [\mathbf{F}(\mathbf{u}) - D\nabla \mathbf{u}] = \mathbf{S}(\mathbf{u}) \quad \text{in } \Omega \times [0, T]$$

which are rewritten as a coupled first order system, e. g., for all $t \in [0, T]$:

$$\begin{aligned} \mathbf{v}_1(\cdot, t) &= \nabla \mathbf{u}(\cdot, t) && \text{in } \Omega, \\ \mathbf{v}_2(\cdot, t) &= \mathbf{S}(\mathbf{u}) - \nabla \cdot [\mathbf{F}(\mathbf{u}(\cdot, t)) - D\mathbf{v}_1(\cdot, t)] && \text{in } \Omega, \\ \partial_t \mathbf{u}(\cdot, t) &= \mathbf{v}_2(\cdot, t) && \text{in } \Omega. \end{aligned}$$

Introduction

Local Discontinuous Galerkin Method

The LDG methods combine advantages from Finite Element and Finite Volume methods such as

- arbitrary unstructured and nonconforming meshes
- higher order approximation spaces
- local conservation properties
- adaptivity in h and p
- well suited for parallel computations

Introduction

LDG for Incompressible Hydrostatic Free Surface Flows

Our aim is to develop a LDG discretization for a *incompressible, hydrostatic free-surface flow* filling a time-dependent domain $\Omega_t \subset \mathbb{R}^d$, $d = 2, 3$.

To overcome the difficulty of the time-dependent domain in the numerical simulation we perform the so-called σ -Transformation. The domain Ω_t is mapped onto a fixed reference domain $\hat{\Omega}$.

- + No mesh adaptation to free surface
- + Higher order approximation for the transformed system
- + Easy implementation
- + Possibly discontinuous height function
- Post-Processing necessary
- Visualization

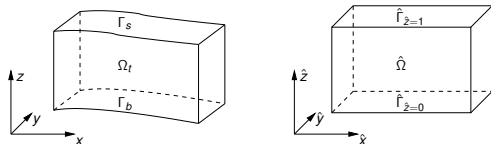


Figure: Time-dependent domain Ω_t and fixed domain after σ -transformation

Incompressible Free Surface Flows

Navier-Stokes Equations and the Hydrostatic System

We consider the d -dimensional incompressible Navier-Stokes equations for shallow flows with a free surface ($d = 2, 3$):

$$\partial_t \mathbf{u}_x + (\mathbf{u} \cdot \nabla) \mathbf{u}_x + \nabla_x p = \mu \Delta \mathbf{u}_x \quad \text{in } \Omega_t,$$

$$\partial_t u_z + (\mathbf{u} \cdot \nabla) u_z + \partial_z p = -g + \mu \Delta u_z \quad \text{in } \Omega_t,$$

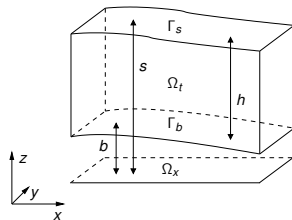
$$\nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_t,$$

$$\partial_t s + \mathbf{u}_x \cdot \nabla_x s = u_z \quad \text{on } \Gamma_s,$$

$$\mathbf{u}_x \cdot \nabla_x b = u_z \quad \text{on } \Gamma_b,$$

$$\kappa \mathbf{u} \cdot \boldsymbol{\tau}_j + \nu \cdot \mathbf{T} \boldsymbol{\tau}_j = 0 \quad \text{on } \Gamma_b,$$

$$\mathbf{T} \nu = 0 \quad \text{on } \Gamma_s.$$



We assume that for all $t \in [0, T]$ the fluid region $\Omega_t \subset \mathbb{R}^d$ has a representation

$$\Omega_t = \left\{ (\mathbf{x}, z) \in \mathbb{R}^d : \mathbf{x} \in \Omega_x, b(\mathbf{x}) < z < s(\mathbf{x}, t) \right\}.$$

$\mathbf{u} = (u_x, u_z)$: velocity
 $\kappa = \text{const}$: friction coefficient

$\mathbf{T} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^\top) - p \mathbb{I}$: total stress tensor

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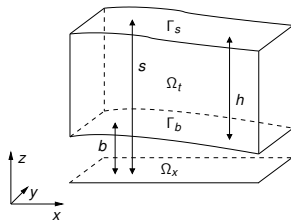
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$$\partial_z p = -g \quad \text{in } \Omega_t,$$

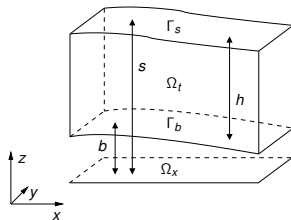
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Incompressible Free Surface Flows

Derivation of the Hydrostatic Model

The above hydrostatic model for shallow incompressible free-surface flows results from a Shallow Water Scaling process as carried out in

J.-F. Gerbeau and B. Perthame. Derivation of Viscous Saint-Venant System for Laminar Shallow Water; Numerical Validation. *Discrete Contin. Dyn. Sys. Ser. B*, 1(1):89-102, 2001.

The evolution in time of the free surface can be computed from

$$\partial_t h + \nabla_x \cdot \left(\int_b^s \mathbf{u}_x \, dz \right) = 0 \quad \text{in } \Omega_x \times [0, T].$$

Solutions to the hydrostatic system may be independent of the vertical coordinate z . In that situation the hydrostatic system coincides with the Shallow Water equations

$$\partial_t \begin{pmatrix} h \\ h\mathbf{u}_x \end{pmatrix} + \nabla_x \cdot \mathbf{F} \begin{pmatrix} h \\ h\mathbf{u}_x \end{pmatrix} = \mathbf{S} \begin{pmatrix} h \\ h\mathbf{u}_x \end{pmatrix} \quad \text{in } \Omega_x \times [0, T],$$

where (in case of $d = 2$)

$$\mathbf{F} \begin{pmatrix} h \\ h\mathbf{u}_x \end{pmatrix} := \begin{pmatrix} h\mathbf{u}_x \\ h\mathbf{u}_x^2 + \frac{1}{2}gh^2 \end{pmatrix}, \quad \mathbf{S} \begin{pmatrix} h \\ h\mathbf{u}_x \end{pmatrix} := \begin{pmatrix} 0 \\ -gh\partial_x b \end{pmatrix}.$$

σ -Transformed Coordinates

General Idea and Notation

Under the assumption of a strictly positive water height we may perform the σ -transformation

$$\hat{t} = t \quad \text{and} \quad (\hat{\mathbf{x}}, \hat{z}) = (\mathbf{x}, \sigma(\mathbf{x}, z, t))$$

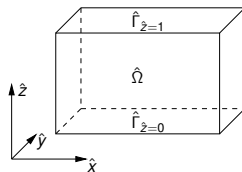
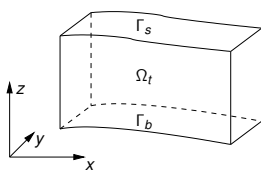
where $\sigma : \overline{\Omega}_t \rightarrow [0, 1]$,

$$\sigma(\mathbf{x}, z, t) := \frac{z - b(\mathbf{x})}{h(\mathbf{x}, t)} \quad (h(\mathbf{x}, t) > 0).$$

Then, for any time $\hat{t} \in [0, \hat{T}]$, we have

$$\hat{\Omega} := \{ (\hat{\mathbf{x}}, \hat{z}) \mid (\mathbf{x}, z) \in \Omega_t \} = \hat{\Omega}_x \times (0, 1),$$

i. e., the transformed domain $\hat{\Omega}$ is fixed in time.



Transformed Hydrostatic System

Transformed Quantities

We express the hydrostatic system in terms of transformed quantities:

- the transformed velocity field $\hat{\mathbf{u}} = (\hat{u}_x, \hat{u}_z)$,

$$\hat{\mathbf{u}}(\hat{\mathbf{x}}, \hat{z}, \hat{t}) := \mathbf{u}(\mathbf{x}, z, t),$$

- the transformed water height and bottom representation

$$\hat{h}(\hat{\mathbf{x}}, \hat{t}) := h(\mathbf{x}, t), \quad \hat{b}(\hat{\mathbf{x}}) := b(\mathbf{x}),$$

for all $(\hat{\mathbf{x}}, \hat{z}) \in \hat{\Omega}$, $\hat{t} \in [0, \hat{T}]$.

Transformed Hydrostatic System

Conservative Formulation

A conservative formulation of the transformed system can be obtained by introducing a function $\hat{\omega} : \hat{\Omega} \times [0, \hat{T}] \rightarrow \mathbb{R}$,

$$\hat{\omega} := \frac{1}{\hat{h}} \left[\hat{u}_z - \hat{\mathbf{u}}_{\mathbf{x}} \cdot \nabla_{\hat{x}} \hat{b} - \hat{z}(\partial_{\hat{t}} \hat{h} + \hat{\mathbf{u}}_{\mathbf{x}} \cdot \nabla_{\hat{x}} \hat{h}) \right] \quad \text{in } \hat{\Omega} \times [0, \hat{T}].$$

- Conservation of mass reads

$$\partial_{\hat{t}} \hat{h} + \nabla \cdot (\widehat{h\mathbf{u}}_{\mathbf{x}}, \widehat{h\omega}) = 0 \quad \text{in } \hat{\Omega}_{\mathbf{x}} \times [0, \hat{T}],$$

- conservation of momentum transforms to

$$\partial_{\hat{t}} (\widehat{h\mathbf{u}}_{\mathbf{x}}) + \nabla \cdot ((\hat{\mathbf{u}}_{\mathbf{x}}, \hat{\omega}) \otimes \widehat{h\mathbf{u}}_{\mathbf{x}}) + \frac{g}{2} \nabla_{\hat{x}} \hat{h}^2 = -g\hat{h} \nabla_{\hat{x}} \hat{b} + \partial_{\hat{z}} \left(\frac{\mu}{\hat{h}} \partial_{\hat{z}} \hat{\mathbf{u}}_{\mathbf{x}} \right) \quad \text{in } \hat{\Omega} \times [0, \hat{T}],$$

- kinematic boundary conditions simplify to

$$\hat{\omega} = 0 \quad \text{on } (\hat{\Gamma}_{\hat{z}=0} \cup \hat{\Gamma}_{\hat{z}=1}) \times [0, \hat{T}].$$

Transformed Hydrostatic System

Closure of the System

In order to close the system in terms of the transformed water height \hat{h} , transformed horizontal momentums $\widehat{h\mathbf{u}_x}$, and $\widehat{h\omega}$, we need an alternative definition of $\widehat{\omega}$.

Because of

$$\partial_{\hat{t}}\hat{h} + \nabla_{\hat{x}} \cdot \left(\int_0^1 \widehat{h\mathbf{u}_x} d\hat{z} \right) = 0 \quad \text{in } \hat{\Omega}_x \times [0, \hat{T}],$$

we can characterize $\widehat{h\omega}$ through the ODE

$$\begin{aligned} \partial_{\hat{z}}(\widehat{h\omega}) &= -\nabla_{\hat{x}} \cdot \left(\widehat{h\mathbf{u}_x} - \int_0^1 \widehat{h\mathbf{u}_x} d\hat{z} \right) && \text{in } \hat{\Omega}_x \times [0, \hat{T}], \\ \widehat{h\omega} &= 0 && \text{on } \hat{\Gamma}_{\hat{z}=0} \times [0, \hat{T}]. \end{aligned}$$

$$\left[\text{cf. } \hat{\omega} = \frac{1}{\hat{h}} \left[\hat{u}_z - \hat{\mathbf{u}}_x \cdot \nabla_{\hat{x}} \hat{b} - \hat{z}(\partial_{\hat{t}}\hat{h} + \hat{\mathbf{u}}_x \cdot \nabla_{\hat{x}}\hat{h}) \right] \right]$$

LDG for the Transformed System

Flux Formulation

The transformed hydrostatic system can be expressed in terms of (*we omit $\hat{\cdot}$*)

$$\mathbf{U}_1 := \left(h, h\mathbf{u}_x \right), \quad \mathbf{U}_2 := \left(\int_0^1 h\mathbf{u}_x dz, h\omega \right).$$

We can rewrite the transformed system in the following form

$$\begin{aligned} \partial_t \mathbf{U}_1 + \nabla \cdot (\mathbf{F}(\mathbf{U}_1, \mathbf{U}_2) - \mathbf{A}(\mathbf{U}_1, \nabla \mathbf{U}_1)) &= \mathbf{S}_1(\mathbf{U}_1), \\ \partial_z \mathbf{U}_2 &= \mathbf{S}_2(\mathbf{U}_1), \end{aligned}$$

where in case of two spatial dimensions the flux, diffusion and source terms are

$$\mathbf{F}(\mathbf{U}_1, \mathbf{U}_2) := \begin{pmatrix} \int_0^1 h\mathbf{u}_x & 0 \\ h\mathbf{u}_x^2 + \frac{1}{2}gh^2 & h\mathbf{u}_x\omega \end{pmatrix}, \quad \mathbf{S}_1(\mathbf{U}_1) := \begin{pmatrix} 0 \\ -gh\partial_x b \end{pmatrix},$$

and

$$\mathbf{A}(\mathbf{U}_1, \nabla \mathbf{U}_1) := \text{diag} \left(0, \frac{\mu}{h} \partial_z \mathbf{u}_x \right), \quad \mathbf{S}_2(\mathbf{U}_1) := \begin{pmatrix} 0 \\ -\partial_x \left(h\mathbf{u}_x - \int_0^1 h\mathbf{u}_x dz \right) \end{pmatrix}.$$

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LDG for the Transformed System

Semi-Discrete Formulation

Let $t \in [0, T]$ be fixed. Given $\mathbf{U}_1(\cdot, t) = (h, h\mathbf{u}_x)(\cdot, t)$ both, the average $\int_0^1 h\mathbf{u}_x(\cdot, t) dz$ and $h\omega(\cdot, t)$, can be computed, i. e.,

$$\mathbf{U}_2(\cdot, t) = \mathcal{L}_{data}[\mathbf{U}_1](\cdot, t),$$

for some suitable spatial operator $\mathcal{L}_{data}[\mathbf{U}_1]$.

We rewrite the transformed hydrostatic system as a first order system

$$\begin{aligned} \mathbf{U}_2(\cdot, t) - \mathcal{L}_{data}[\mathbf{U}_1](\cdot, t) &= 0, \\ \mathbf{V}_1(\cdot, t) - \mathbf{A}(\mathbf{U}_1, \nabla \mathbf{U}_1)(\cdot, t) &= 0, \\ \mathbf{V}_2(\cdot, t) - \nabla \cdot \mathbf{V}_1(\cdot, t) &=: \mathbf{V}_2(\cdot, t) - \mathcal{L}_{impl}[\mathbf{U}_1](\cdot, t) = 0, \\ \mathbf{V}_3(\cdot, t) + \nabla \cdot \mathbf{F}(\mathbf{U}_1, \mathbf{U}_2)(\cdot, t) - \mathbf{S}_1(\mathbf{U}_1)(\cdot, t) &=: \mathbf{V}_3(\cdot, t) - \mathcal{L}_{expl}[\mathbf{U}_1, \mathbf{U}_2](\cdot, t) = 0, \\ \partial_t \mathbf{U}_1(\cdot, t) - \underbrace{\mathcal{L}_{expl}[\mathbf{U}_1, \mathbf{U}_2](\cdot, t) - \mathcal{L}_{impl}[\mathbf{U}_1](\cdot, t)}_{=: \mathcal{L}_{expl} \circ \mathcal{L}_{data}[\mathbf{U}_1](\cdot, t)} &= 0. \end{aligned}$$

This system can be integrated in time via Semi-Implicit Runge-Kutta Methods (SIRK, IMEX, etc.).

LDG for the Transformed System

Discrete Problem

Choose a grid $\mathcal{T}_{\mathcal{N}}$ for $\Omega \subset \mathbb{R}^d$, $d = 2, 3$. Let

$$\begin{array}{ll}
 t = 0 & \text{start time,} \\
 h_{\mathcal{N}}(\cdot, 0) & \text{initial discrete transformed water height,} \\
 (h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}}(\cdot, 0) & \text{initial discrete transformed horizontal momentum.}
 \end{array}$$

While ($t < T$):

1. compute the averaged horizontal momentums $\int_0^1 (h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}}(\cdot, t) dz$,
2. compute $\partial_z(h\omega)_{\mathcal{N}}(\cdot, t) = -\partial_x \left((h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}} - \int_0^1 (h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}} dz \right)$,
3. compute updates $\Delta h_{\mathcal{N}}, \Delta(h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}}, \Delta t$,
4. set

$$\begin{aligned}
 h_{\mathcal{N}}(\cdot, t + \Delta t) &= h_{\mathcal{N}}(\cdot, t) + \Delta h_{\mathcal{N}}, \\
 (h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}}(\cdot, t + \Delta t) &= (h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}}(\cdot, t) + \Delta(h\mathbf{u}_{\mathbf{x}})_{\mathcal{N}},
 \end{aligned}$$

5. set $t = t + \Delta t$.

DUNE

Modular Interface Library



<http://dune-project.org/>

DUNE (*Distributed and Unified Numerics Environment*) is a modular interface library developed by groups in Heidelberg, Berlin, Freiburg and Münster.

Dune-Grid

- abstract parallel grid interface definition
- implementations: ALBERTA, ALUGrid, UG, YaspGrid, ...

Dune-Fem

- abstract parallel discretization interface definitions
- discretization schemes (LDG, Finite Volumes)
- ODE solvers ...

Dune-PrismGrid

- new DUNE grid implementation
- generic prismatic elements over arbitrary DUNE grids

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Dune-PrismGrid

Generic Prismatic Grid

Design Principles

- Implementation in DUNE
- Generic prismatic elements over arbitrary n -dimensional DUNE grid
- Structured in vertical direction
- Several additional iterators for columns and layers
- Suitable for parallel computations

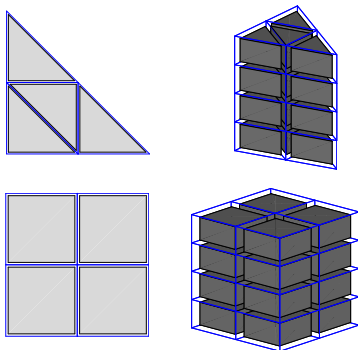


Figure: 2d host grids and resulting 3d prismatic grids

Dune-PrismGrid

Generic Prismatic Grid

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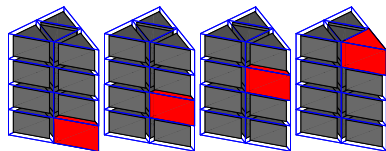


Figure: Iteration upwards a Column

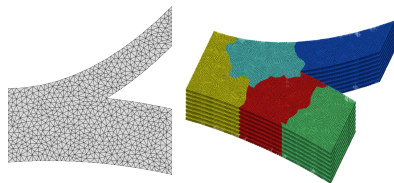


Figure: 2d macro grid, resulting 3d prismatic grid with 5 partitions

Numerical Results

Quasi-Riemann Problem

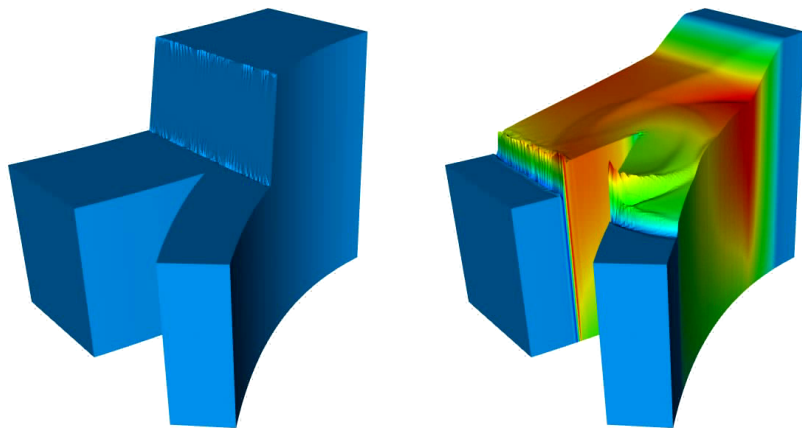


Figure: (Left) 3D representation of initial conditions and (Right) solution to a latter time

Numerical Results

Quasi-Riemann Problem

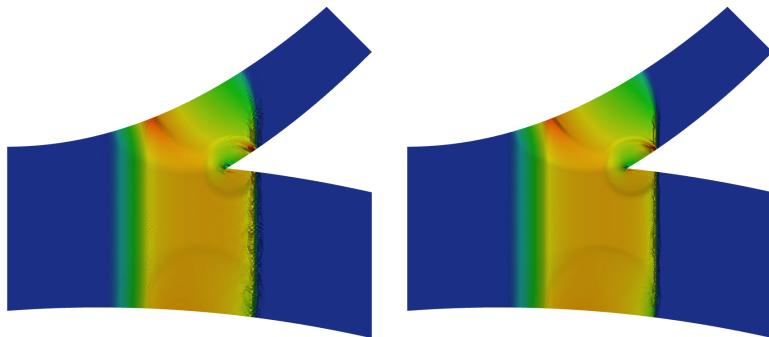


Figure: Numerical solution at time $t \approx 0.15$: (Left) Coarser grid with 917760 nodes and (Right) finer grid with 3671040 nodes, 8 layers each, piecewise linear ansatz functions

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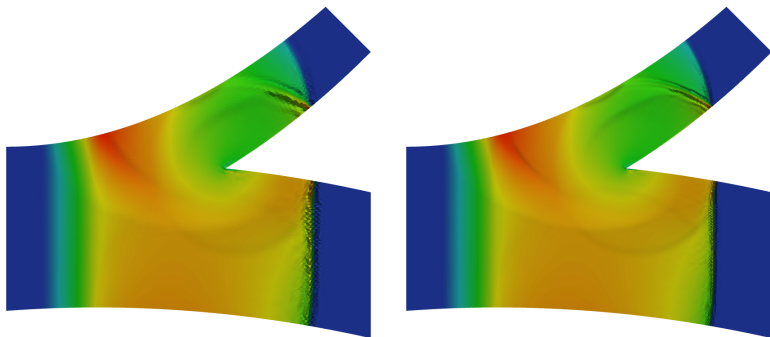


Figure: Numerical solution at time $t \approx 0.30$: (Left) Coarser grid with 917760 nodes and (Right) finer grid with 3671040 nodes, 8 layers each, piecewise linear ansatz functions

Conclusion

Summary and Remarks

- We presented a model for shallow incompressible, hydrostatic flows with a free surface,
- and developed a Local Discontinuous Galerkin discretization for the transformed system in σ -coordinates.
- Higher order approximation in space (*polynomial degree of the DG-space*) and time (*Higher Order Explicit-Implicit Runge Kutta method*).
- No smoothing of the discrete height function needed.
- Generic parallel prismatic grids conforming to discretization requirements.

Thank you for your attention.

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